Special Relativity Cheat Sheet



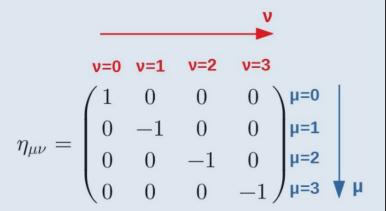
Spacetime interval

$$\begin{split} (dS)^2 &= \eta_{\mu\nu} dx^{\mu} dx^{\nu} \\ &= c^2 (dt)^2 - (dx)^2 - (dy)^2 - (dz)^2 \end{split}$$

Proper time interval

$$\begin{split} d\tau &= \frac{1}{c}\sqrt{\eta_{\mu\nu}dx^{\mu}dx^{\nu}}\\ &= \frac{1}{c}\sqrt{c^2(dt)^2-(dx)^2-(dy)^2-(dz)^2} \end{split}$$

Minkowski metric



Lorentz transformations (x-direction)

Alternate form for spacetime and proper time intervals

$$dS = \frac{1}{\gamma}cdt$$

$$d\tau = \frac{1}{\gamma}dt$$

$$x'=\gamma\left(x-vt\right)$$
 , where $\gamma=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, $t'=\gamma\left(t-\frac{v}{c^2}x\right)$

Important 4-Vector Quantities:

4-position

$$x^{\mu} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \begin{matrix} \mu=0 \\ \mu=1 \\ \mu=2 \\ \mu=3 \end{matrix}$$

4-gradient operator

4-velocity

$$u^{\mu} = \left(\begin{matrix} \gamma c \\ \gamma v \end{matrix} \right)_{\text{μ=1,2,3}}^{\text{μ=0}}$$

4-momentum

$$p^{\mu}=\left(rac{E}{c}
ight)_{\mu=1,2,3}^{\mu=0}$$

4-force

4-force
$$F^{\mu} = \begin{pmatrix} \frac{1}{c} \frac{dE}{d\tau} \\ \frac{dp}{d\tau} \end{pmatrix} = \begin{pmatrix} \frac{1}{c} \gamma \frac{dE}{dt} \\ \gamma \frac{dp}{dt} \end{pmatrix} \underset{\mu=1,2,3}{\mu=0}$$

$$\mu=0$$

$$\mu=1,2,3$$

$$dW^{\mu} = \begin{pmatrix} dW \\ cdI_x \\ cdI_y \\ cdI_z \end{pmatrix} \underset{\mu=2}{\mu=2}$$

$$\mu=3$$
 Electromagnetic 4-

$$dW^{\mu} = \begin{pmatrix} dW \\ cdI_x \\ cdI_y \\ cdI_z \end{pmatrix} \begin{array}{l} \mu = 0 \\ \mu = 1 \\ \mu = 2 \\ \mu = 3 \end{array}$$

Electromagnetic 4potential

$$A^{\mu}=egin{pmatrix} rac{arphi}{c} \ A_x \ A_y \ A_z \end{pmatrix} egin{pmatrix} \mu=0 \ \mu=1 \ \mu=2 \ \mu=3 \ \end{pmatrix}$$

Other Important Stuff:

Relativistic total energy

$$E = \gamma mc^{2}$$

$$= mc^{2} + \frac{1}{2}mv^{2} + \frac{3}{8}m\frac{v^{4}}{c^{2}} + \frac{5}{16}m\frac{v^{6}}{c^{4}} + \dots$$

Energy associated with mass (rest energy)

Ordinary Newtonian kinetic energy

Relativistic kinetic energy terms (only come into play at speeds close to c)

Relativistic Lagrangian (for a free particle)

$$\mathcal{L} = -\frac{1}{\gamma}mc^2$$

Relativistic kinetic energy

$$E_k = mc^2 \left(\gamma - 1 \right)$$

Electromagnetic field tensor

$$F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} = \begin{pmatrix} 0 & -\frac{E_{x}}{c} & -\frac{E_{y}}{c} & -\frac{E_{z}}{c} \\ \frac{E_{x}}{c} & 0 & -B_{z} & B_{y} \\ \frac{E_{y}}{c} & B_{z} & 0 & -B_{x} \\ \frac{E_{z}}{c} & -B_{y} & B_{x} & 0 \end{pmatrix} \begin{matrix} \mu=0 \\ \mu=1 \\ \mu=2 \\ \mu=3 \end{matrix} \downarrow \mu$$

Lorentz Invariant Quantities + Relativistic Laws Of Motion:

Lorentz invariant quantities can be constructed by multiplying a contravariant vector (A^µ) with its covariant version (A_µ):

$$A_{\mu}A^{\mu} = \left(A^{0}\right)^{2} - \left(A^{1}\right)^{2} - \left(A^{2}\right)^{2} - \left(A^{3}\right)^{2}$$

Spacetime interval + the speed of light

$$x_{\mu}x^{\mu} = S^2$$

$$u_{\mu}u^{\mu} = c^2$$

Relativistic work-energy theorem

$$dW_{\mu}dW^{\mu} = c^2 dp_{\mu} dp^{\mu}$$

Relativistic Lorentz force law

$$F^{\mu} = q u_{\nu} F^{\mu\nu}$$

Momentum-energy relation

$$p_{\mu}p^{\mu} = m^2c^2 \quad \Rightarrow \quad E^2 = m^2c^4 + p^2c^2$$