Special Relativity Cheat Sheet

PROFOUNDPHYSICS

Spacetime interval

$$
\begin{aligned}
(dS)^2 &= \eta_{\mu\nu} dx^{\mu} dx^{\nu} \\
&= c^2 (dt)^2 - (dx)^2 - (dy)^2 - (dz)^2
\end{aligned}
$$

Proper time interval

$$
\begin{aligned}
d\tau &= \frac{1}{c}\sqrt{\eta_{\mu\nu} dx^{\mu} dx^{\nu}} \\
&= \frac{1}{c}\sqrt{c^2 (dt)^2 - (dx)^2 - (dy)^2 - (dz)^2}
\end{aligned}
$$

Minkowski metric

$$
\xrightarrow{\nu}
$$

$$
\eta_{\mu\nu} = \begin{array}{c}
\begin{array}{cccc} \nu=0 & \nu=1 & \nu=2 & \nu=3 \end{array} \\
\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}
\end{array}
\begin{array}{c} \mu=0 \\ \mu=1 \\ \mu=2 \\ \mu=3 \end{array}
\Big\downarrow \mu
$$

Alternate form for spacetime and proper time intervals

$$
dS = \frac{1}{\gamma} c\,dt
$$

$$
d\tau = \frac{1}{\gamma} dt
$$

Lorentz transformations
(x-direction)

$$
\begin{aligned}
x' &= \gamma (x - vt) \\
t' &= \gamma \left(t - \frac{v}{c^2} x \right)
\end{aligned}
\quad \text{, where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
$$

Important 4-Vector Quantities:

4-position

$$x^\mu = \begin{array}{cl} \left(\begin{array}{c} ct \\ x \\ y \\ z \end{array}\right) & \begin{array}{l} \mu=0 \\ \mu=1 \\ \mu=2 \\ \mu=3 \end{array} \end{array}$$

4-gradient operator

$$\partial_\mu = \begin{array}{cl} \left(\begin{array}{c} \frac{1}{c}\frac{\partial}{\partial t} \\ \vec{\nabla} \end{array}\right) & \begin{array}{l} \mu=0 \\ \mu=1,2,3 \end{array} \end{array}$$

4-velocity

$$u^\mu = \begin{array}{cl} \left(\begin{array}{c} \gamma c \\ \gamma v \end{array}\right) & \begin{array}{l} \mu=0 \\ \mu=1,2,3 \end{array} \end{array}$$

4-momentum

$$p^\mu = \begin{array}{cl} \left(\begin{array}{c} \frac{E}{c} \\ \gamma m v \end{array}\right) & \begin{array}{l} \mu=0 \\ \mu=1,2,3 \end{array} \end{array}$$

4-force

$$F^\mu = \left(\begin{array}{c} \frac{1}{c}\frac{dE}{d\tau} \\ \frac{dp}{d\tau} \end{array}\right) = \begin{array}{cl} \left(\begin{array}{c} \frac{1}{c}\gamma\frac{dE}{dt} \\ \gamma\frac{dp}{dt} \end{array}\right) & \begin{array}{l} \mu=0 \\ \mu=1,2,3 \end{array} \end{array}$$

4-impulse / 4-work

$$dW^\mu = \begin{array}{cl} \left(\begin{array}{c} dW \\ cdI_x \\ cdI_y \\ cdI_z \end{array}\right) & \begin{array}{l} \mu=0 \\ \mu=1 \\ \mu=2 \\ \mu=3 \end{array} \end{array}$$

Electromagnetic 4-potential

$$A^\mu = \begin{array}{cl} \left(\begin{array}{c} \frac{\varphi}{c} \\ A_x \\ A_y \\ A_z \end{array}\right) & \begin{array}{l} \mu=0 \\ \mu=1 \\ \mu=2 \\ \mu=3 \end{array} \end{array}$$

Other Important Stuff:

Relativistic total energy

$$E = \gamma mc^2$$
$$= \underbrace{mc^2}_{\substack{\text{Energy}\\\text{associated}\\\text{with mass}\\\text{(rest energy)}}} + \underbrace{\frac{1}{2}mv^2}_{\substack{\text{Ordinary}\\\text{Newtonian}\\\text{kinetic energy}}} + \underbrace{\frac{3}{8}m\frac{v^4}{c^2} + \frac{5}{16}m\frac{v^6}{c^4} + \dots}_{\substack{\text{Relativistic}\\\text{kinetic energy}\\\text{terms (only}\\\text{come into play}\\\text{at speeds}\\\text{close to c)}}}$$

Relativistic Lagrangian (for a free particle)

$$\mathcal{L} = -\frac{1}{\gamma}mc^2$$

Relativistic kinetic energy

$$E_k = mc^2(\gamma - 1)$$

Electromagnetic field tensor

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{array}{c} \nu \rightarrow \\ \begin{array}{cccc} \nu=0 & \nu=1 & \nu=2 & \nu=3 \end{array} \\ \begin{pmatrix} 0 & -\frac{E_x}{c} & -\frac{E_y}{c} & -\frac{E_z}{c} \\ \frac{E_x}{c} & 0 & -B_z & B_y \\ \frac{E_y}{c} & B_z & 0 & -B_x \\ \frac{E_z}{c} & -B_y & B_x & 0 \end{pmatrix} \end{array} \begin{array}{l} \\ \\ \mu=0 \\ \mu=1 \\ \mu=2 \\ \mu=3 \; \downarrow \mu \end{array}$$

Lorentz Invariant Quantities + Relativistic Laws Of Motion:

Lorentz invariant quantities can be constructed by multiplying a contravariant vector (A^μ) with its covariant version (A_μ):

$$A_\mu A^\mu = \left(A^0\right)^2 - \left(A^1\right)^2 - \left(A^2\right)^2 - \left(A^3\right)^2$$

Spacetime interval + the speed of light

$$x_\mu x^\mu = S^2$$

$$u_\mu u^\mu = c^2$$

Relativistic work-energy theorem

$$dW_\mu dW^\mu = c^2 dp_\mu dp^\mu$$

Relativistic Lorentz force law

$$F^\mu = q u_\nu F^{\mu\nu}$$

Momentum-energy relation

$$p_\mu p^\mu = m^2 c^2 \quad \Rightarrow \quad E^2 = m^2 c^4 + p^2 c^2$$